Transformada de Fourier
de uma função exponencial

- Prof. Nilton Cesar de Oliveira Borges

$$f(t) = e^{-at} \qquad\qquad F(\omega) = \int_{-\infty}^{\infty} e^{-at} \cdot e^{-j\omega \cdot t} dt$$

$$F(\omega) = \int_{0}^{\infty} e^{-(a+j\omega)t} dt \qquad\qquad -\left[\frac{1}{a+j\omega} \cdot e^{-(a+j\omega)t} \right]_{0}^{\infty}$$

$$-\left[\frac{1}{a+j\omega} \cdot e^{-(a+j\omega)\infty} - \frac{1}{a+j\omega} \cdot e^{-(a+j\omega)\cdot 0} \right]$$

$$-\left[\frac{1}{a+j\omega} \cdot e^{-(a+j\omega)\infty} \xrightarrow{} \mathbf{0} - \frac{1}{a+j\omega} \cdot e^{-(a+j\omega)\cdot 0} \xrightarrow{} \mathbf{1} \right]$$

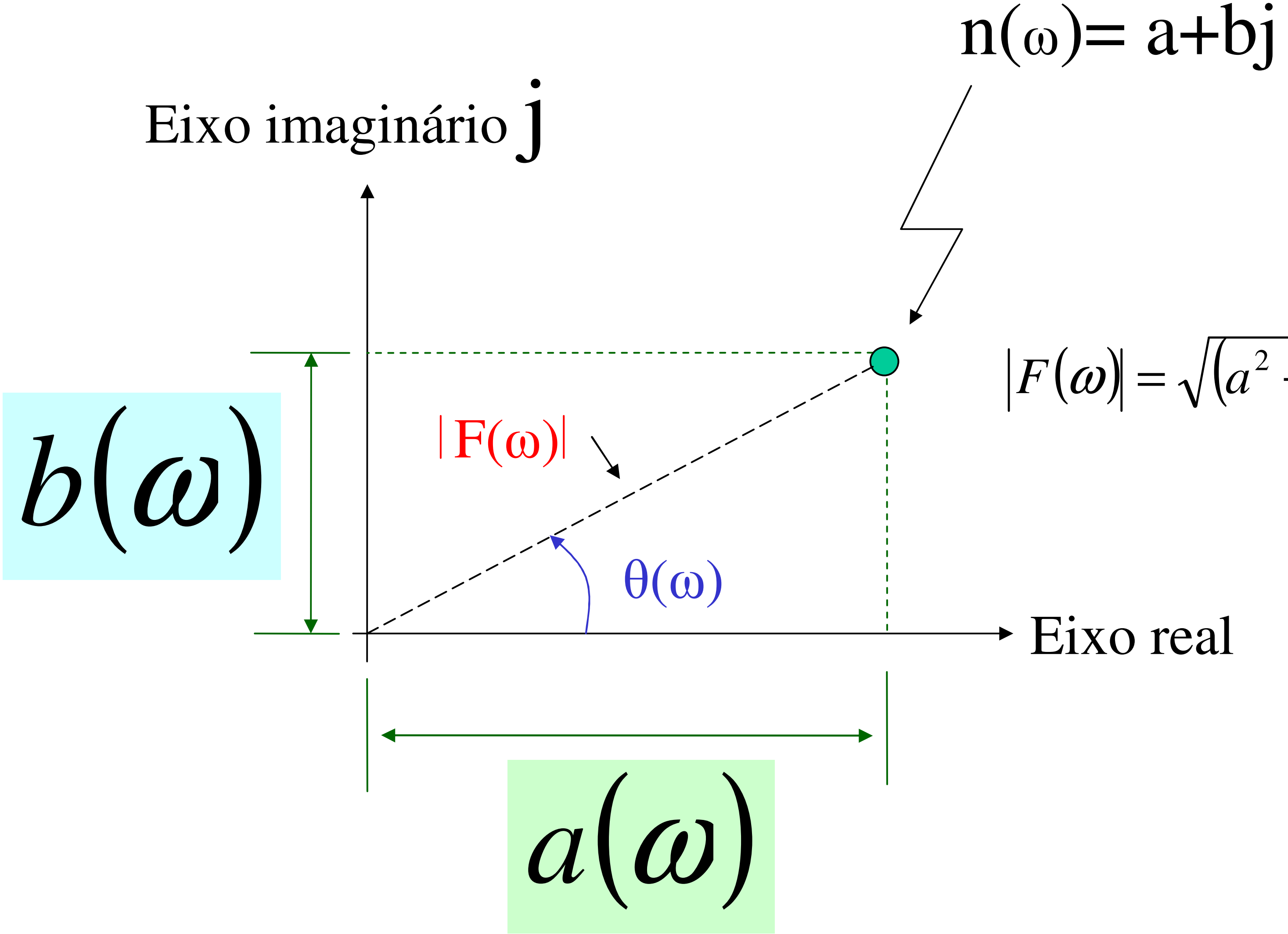
$$\frac{1}{a+j\omega}$$

$$\frac{1}{a+j\omega}\cdot\frac{a-j\omega}{a-j\omega}\Rightarrow\frac{a-j\omega}{a^2+\omega^2}$$

$$\frac{a-j\omega}{a^2+\omega^2}$$

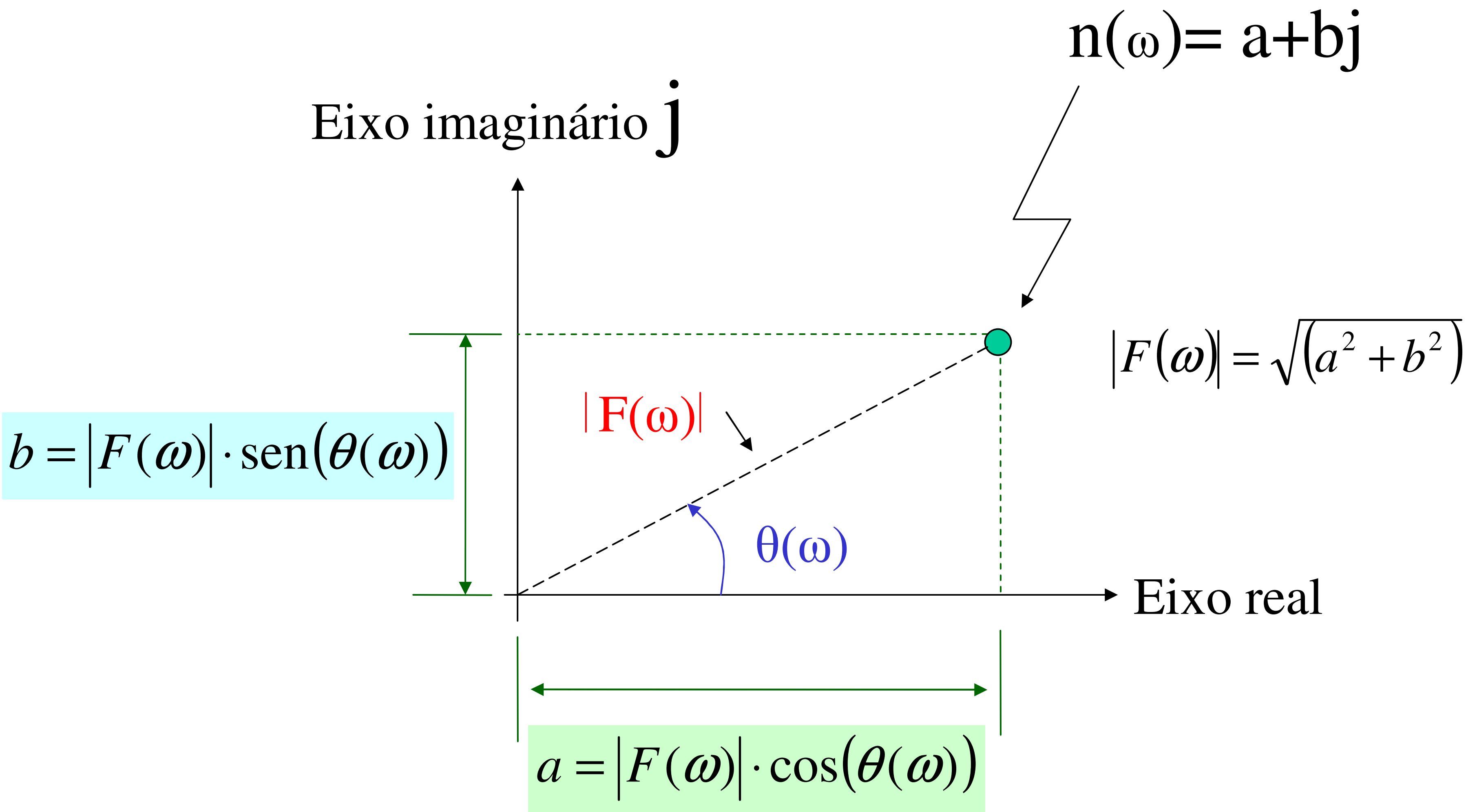
Deve-se lembrar se trata de uma função complexa que pode também ser escrita na forma de **módulo** e fase, sendo ambos em função de ω.

Para escrevermos uma função complexa vamos recordar, tendo um número n em função de ω



$$|F(\omega)| = \sqrt{(a^2 + b^2)}$$

Para escrevermos uma função complexa vamos recordar, tendo um número n em função de ω



Portanto a função fica como:

$$|F(\omega)| \cdot \cos(\theta(\omega)) + j.|F(\omega)| \cdot \text{sen}(\theta(\omega))$$

Colocando F(ω) em evidência temos:

$$|F(\omega)| \cdot [\cos(\theta(\omega)) + j \cdot \text{sen}(\theta(\omega))]$$

Utilizando a identidade de EULER a forma final da função é:

$$|F(\omega)| \cdot e^{j\theta(\omega)}$$

Logo a transformada calculada ficará:

$$\frac{a - j\omega}{a^2 + \omega^2} \cdot \frac{a + j\omega}{a + j\omega} \Leftrightarrow \frac{a - j\omega}{a^2 + \omega^2}$$

$$\xrightarrow{\text{temos então o módulo igual a}} \left|F(\omega)\right| = \frac{\sqrt{a^2 + \omega^2}}{a^2 + \omega^2}$$

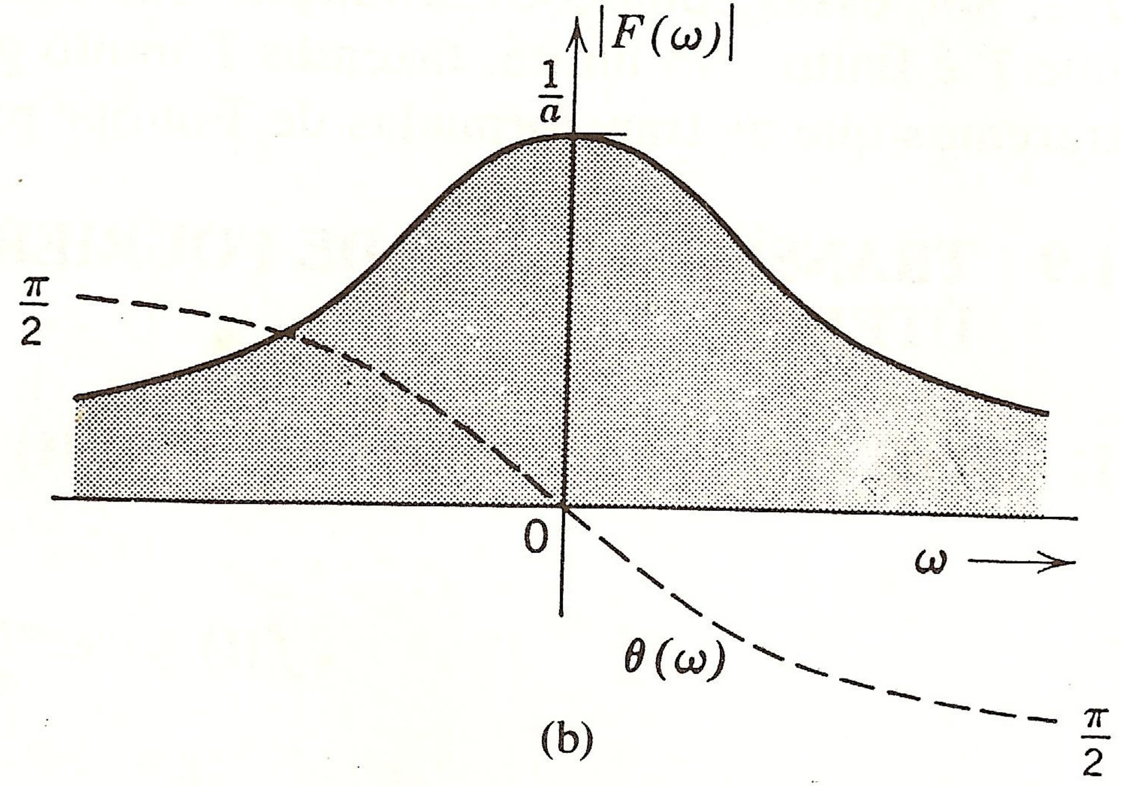
$$\left|F(\omega)\right| = \frac{1}{\sqrt{a^2 + \omega^2}}$$

$$\xrightarrow{\text{e a fase igual a}} |\theta(\omega)| = -arctag\left(\frac{\omega}{a}\right)$$

Sendo então a forma final da função como:

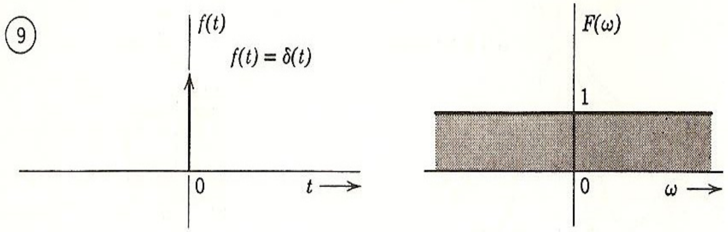
$$\frac{1}{\sqrt{a^2+\omega^2}} \cdot e^{-j.arctag\left(\frac{\omega}{a}\right)}$$

O gráfico da transformada de fourier
fica então:

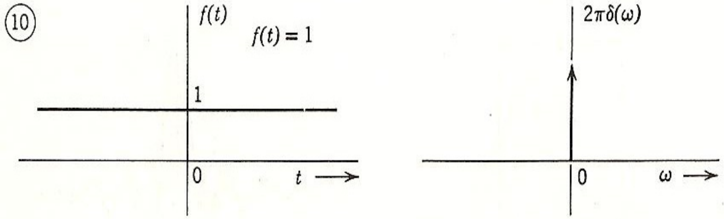


(b)

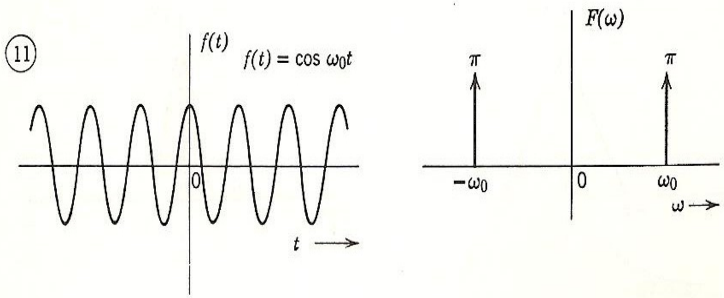
(9)



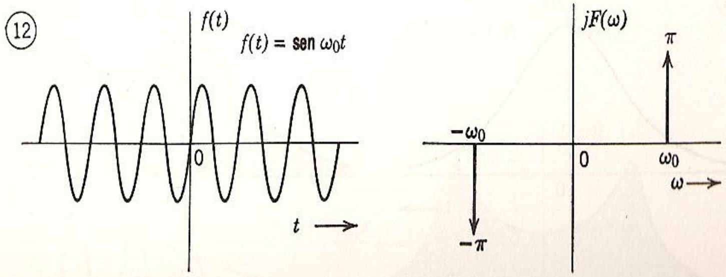
(10)



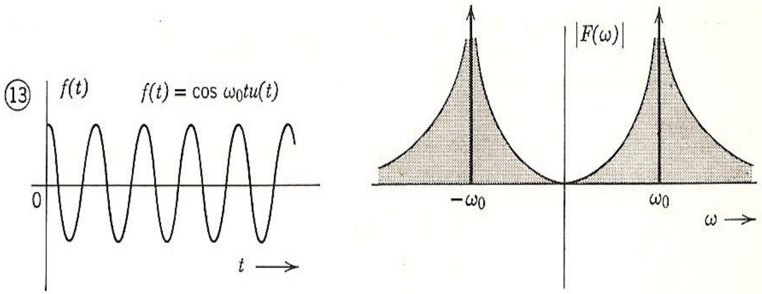
(11)



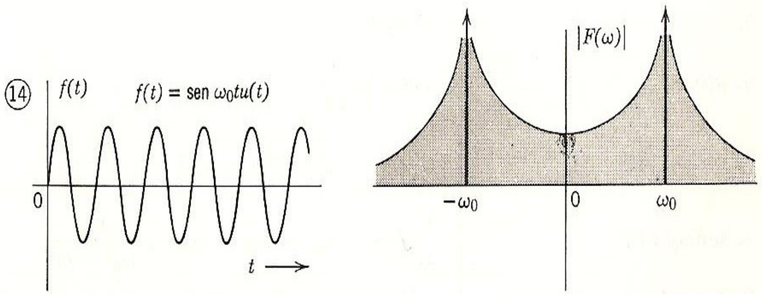
(12)



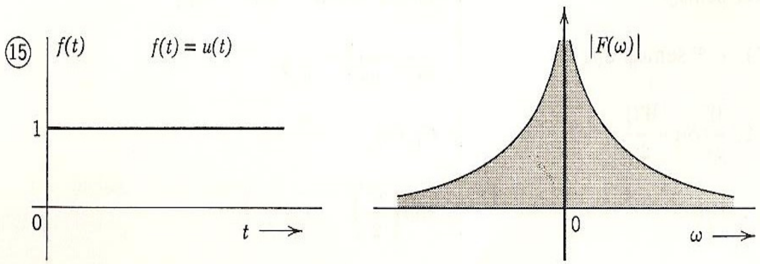
(13)



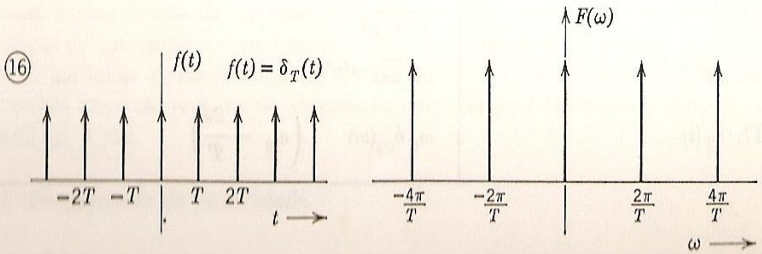
(14)



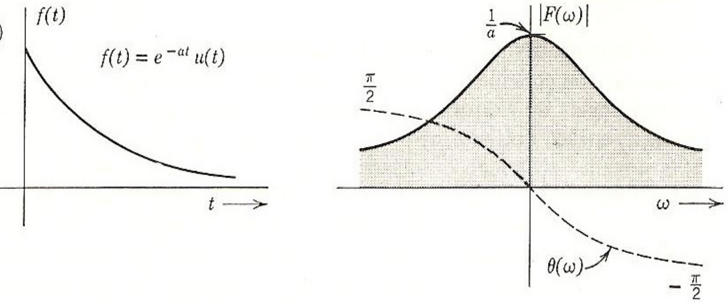
(15)



(16)

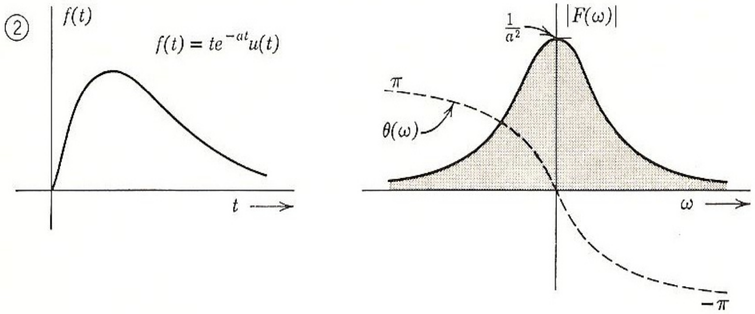


①



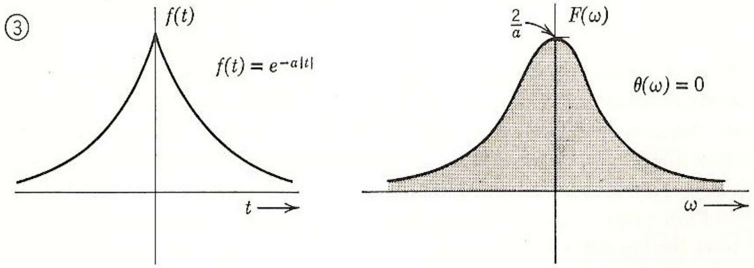
f(t)
f(t) = e^{-at} u(t)
t
|F(ω)|
1/a
π/2
ω
θ(ω)
−π/2

②



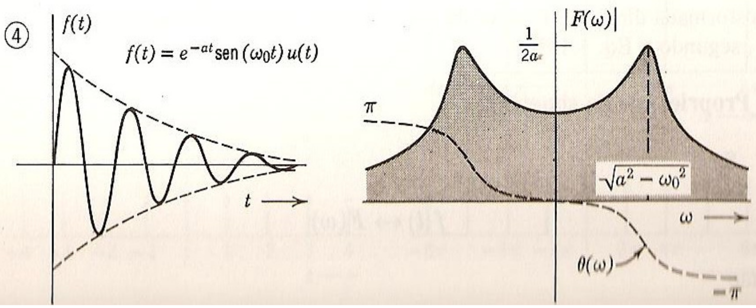
f(t)
f(t) = te^{-at}u(t)
t
|F(ω)|
1/a²
π
θ(ω)
ω
−π

③



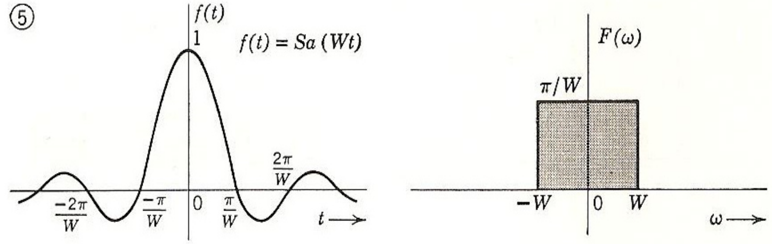
f(t)
f(t) = e^{-a|t|}
t
F(ω)
2/a
θ(ω) = 0
ω

④



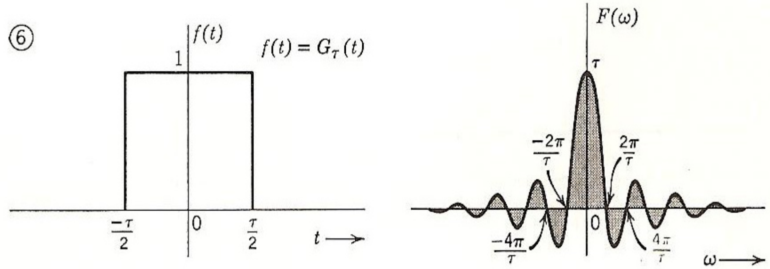
f(t)
f(t) = e^{-at} sen (ω₀t) u(t)
t
|F(ω)|
π
√(a² − ω₀²)
ω
θ(ω)
−π

⑤



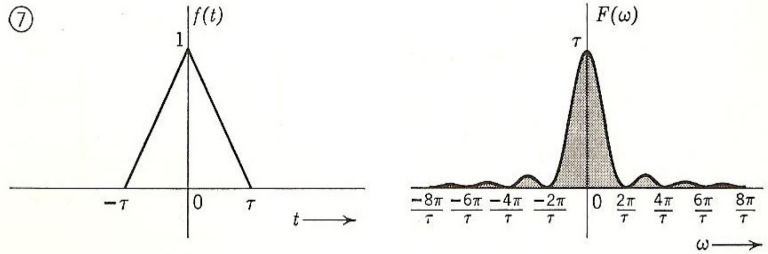
f(t)
1
f(t) = Sa (Wt)
−2π/W
−π/W
0
π/W
2π/W
t
F(ω)
π/W
−W
0
W
ω

⑥



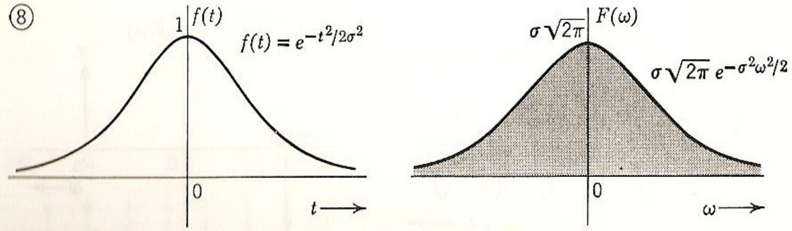
f(t)
1
f(t) = G_τ (t)
−τ/2
0
τ/2
t
F(ω)
τ
−2π/τ
2π/τ
−4π/τ
0
4π/τ
ω

⑦



f(t)
1
−τ
0
τ
t
F(ω)
τ
−8π/τ
−6π/τ
−4π/τ
−2π/τ
0
2π/τ
4π/τ
6π/τ
8π/τ
ω

⑧



f(t)
1
f(t) = e^{−t²/2σ²}
0
t
F(ω)
σ√2π
σ√2π e^{−σ²ω²/2}
0
ω

Tabela I.IB Transformadas de Fourier

$f(t)$	$F(\omega)$
1. $e^{-at}u(t)$	$\dfrac{1}{a + j\omega}$
2. $te^{-at}u(t)$	$\dfrac{1}{(a + j\omega)^2}$
3. $\lvert t\rvert$	$\dfrac{-2}{\omega^2}$
4. $\delta(t)$	1
5. 1	$2\pi\,\delta(\omega)$
6. $u(t)$	$\pi\,\delta(\omega) + \dfrac{1}{j\omega}$
7. $\cos\omega_0 t\; u(t)$	$\dfrac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \dfrac{j\omega}{\omega_0^2 - \omega^2}$
8. $\operatorname{sen}\omega_0 t\; u(t)$	$\dfrac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \dfrac{\omega_0}{\omega_0^2 - \omega^2}$
9. $\cos\omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
10. $\operatorname{sen}\omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$
11. $e^{-at}\operatorname{sen}\omega_0 t\; u(t)$	$\dfrac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$
12. $\dfrac{W}{2\pi} Sa\dfrac{(Wt)}{2}$	$G_W(\omega)$
13. $G_\tau(t)$	$\tau Sa\left(\dfrac{\omega\tau}{2}\right)$
14. $\begin{cases} 1 - \dfrac{\lvert t\rvert}{\tau} & \cdots \lvert t\rvert < \tau \\ 0 & \cdots \lvert t\rvert > \tau \end{cases}$	$\tau\left[Sa\left(\dfrac{\omega\tau}{2}\right)\right]^2$
15. $e^{-a\lvert t\rvert}$	$\dfrac{2a}{a^2 + \omega^2}$
16. $e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$
17. $\delta_T(t)$	$\omega_0\,\delta_{\omega_0}(\omega) \qquad \left(\omega_0 = \dfrac{2\pi}{T}\right)$